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APPLICATION OF COMSOL MULTIPHYSCIS TO SIMULATE STEADY NATURAL CONVECTION IN THE AIR AROUND A HOT WATER PIPE

[1]BELLO M. A., [2]OLADIMEJI S. R.

[1,2]DEPARTMENT OF TECHNICAL EDUCATION

OSUN STATE COLLEGE OF EDUCATION, ILA-ORANGUN

OSUN STATE, NIGERIA

Corresponding author: engrbellomoruff@gmail.com

ABSTRACT: *COMSOL Multiphysics has been devised and developed to enable the realistic and accurate modelling and simulation of the behaviour and performance of components of engineering systems. In the present work, the steady flow of hot water in a straight horizontal carbon-steel pipe located in cool quiescent air is modelled. As such, forced convection in the internal flow, conduction through the pipe's material and natural convection in the ensuing external flow were simultaneously simulated. Water entered the pipe at 90°C and leaves it at 80°C whilst the ambient air is at 25°C. Three different modelling techniques were adopted. The first involved only the natural convection in the air, the second combined the conduction through the pipe with the natural convection in the air whilst the third technique included the internal forced convection. For each of the first two techniques, the two case studies considered assumed that the internal flow is fully-developed; one case each for laminar (Re = 500) and turbulent (Re = 2×10^4) regimes. For the last two case studies, the internal flow was each implemented as developing laminar or turbulent flow for which the length of the pipe was appropriately prescribed to be 1.627m. Fully-developed natural convection was maintained by density drops of the air of up to 15% which resulted in air speeds exceeding 0.3m/s around the pipe's surface and 0.5m/s above it with convective heat transfer coefficients in the range of 7 to 10W/m^2.K. The internal forced convection became fully-developed between 5.81mm and 1.72mm pipe diameters downstream of the entrance for Re of 500 and $2x10^3$ respectively with convective heat transfer coefficients predicted to be in the range of 9 to 11W/m^2. K.*

Keywords: *Convection, Conduction, COMSOL Heat transfer coefficient, Multiphysics, Nusselt number*

1.0 INTRODUCTION

Convective heat transfer is the heat transfer process that arises from fluid flow over or through a hot or cold surface where the fluid flow acts as a carrier of energy. The fluid heating or cooling process is called natural or free convection if the fluid motion is induced only by the buoyancy forces and forced convection if the fluid motion is instigated by some devices like fan, pump etc.

Natural convection in fluids occurs when buoyancy forces are induced from the density changes caused by the temperature gradient in the fluid region close to the heat transfer surface. In other words, a fluid attached to a hot or cold surface will change its temperature causing a drop in its density in the vicinity of that surface, initiating it to move up- or downward, depending on whether the fluid is heated or cooled. The principles of natural convection flow around heated, horizontal, inclined and vertical pipes (or cylinders) have been extensively applied in many technologies. These applications are commonly encountered in cooling of automotive engines, rockets, jet engines, nuclear power plants, solar collectors, in transportation of water, crude oil and natural gas. It is also, encountered in a converter used for space heating, where horizontal tubes are placed in an enclosure with openings at the top and bottom to allow the passage of air through the vertical channel and in the air cooling of electronic components in vented enclosures in many electronic systems.

Modelling the laminar natural convection taking place in the air that surrounds a metal pipe of specified inner and outer diameters in which hot water is flowing with a specified mass flow rate using COMSOL Multiphysics Software will go a long way in ensuring the effective operations of many modern engineering components. Thus, the objectives of this research work are to:

i. Analyze with COMSOL Multiphysics software the two mode of heat transfer i.e. conduction in the pipe material, and both internal and external convection in and out of the pipe respectively.
ii. Study the velocity, temperature and density distributions in the air surrounding the metal pipe, and;
iii. Find the estimates of the average value of the convective heat transfer coefficient and the Nusselt number around the pipe and compare it with prevailing physical experimental work.

The physics of the problem is limited to that of external natural convection, internal flow convection and conduction in the pipe material while radiation from the external surface of the pipe is not considered. The temperature of the flowing water is used to determine the temperature boundary condition at the interior surface of the pipe for heat conduction through the pipe material and the temperature boundary condition at the pipe's exterior surface for natural convection in the air.

Fluid flow over pipes is commonly used in heating and cooling applications. The fluid in such applications is allowed to flow by a fan or pump or by natural means inside a tube that is adequately long to accomplish the desired heat transfer.

2.0 LITERATURE REVIEW

2.1 Heat Conduction in Pipe Material

In the recent past, an extensive literature review on general heat transfer has been done by many researchers where works related to the science of heat transfer, including numerical, analytical and experimental works were reviewed. For example, Cengel (2003), experiments have shown that the rate of heat transfer through the pipe wall is doubled when the temperature difference across the wall or the area normal to the direction of heat transfer is doubled, but is halved when the pipe wall thickness is doubled.The ability of the pipe material to conduct heat is the thermal conductivity.

Similarly, Ozisik (1993) also proved that thermal conductivity is a very important thermo-physical property of a material because the heat flux increases with increasing thermal conductivity under a prescribed temperature gradient. Since the interaction between molecules in the solid is strongest, the thermal conductivity for the solid is usually higher than that of the liquid and gas. To understand the parameters affecting conduction, an experimental study of the heat transfer coefficient associated with solid-liquid mixture transport was carried out by Rozenblit, Simkhis, Hetsroni, Barnea and Taitel (2000), using an electro-resistance sensor and infra-red imaging and they discovered that the average heat transfer increases with particle concentration. Their work also revealed that local value of the heat transfer coefficient is strongly influenced by the cross sectional distribution of the solid phase in the pipe. According to Faghri et al., (2010), the heat conduction general equation in pipe material is given as

$$\frac{1}{\mathrm{r}}\frac{\partial}{\partial \mathrm{r}}\left\{\mathrm{kr}\frac{\partial \mathrm{T}}{\partial \mathrm{r}}\right\} = 0 \quad (1)$$

Where r is the radial distance from the centre of the pipe, k is the thermal conductivity and T is the temperature variation along the pipe material and depends on r.

2.2 Internal Convective Heat Flow in Pipe

Different inlet geometries for laminar air flow combined convection heat transfer inside a horizontal circular pipe, through careful measurements, was experimentally studied by Hussein (2009) under a constant wall heat flux boundary condition. It was found that the surface temperature values along the pipe dimensionless axial distance were higher for low Reynolds number than that for high Reynolds number due to the free convection domination on the heat transfer process.

2.0 Heat Transfer from Hot Water

When hot water flows through a pipe, heat will be transferred from the surface of the pipe to the external air. For constant surface temperature, fully developed laminar flow in pipe, Nusselt number is given as 3.66 and 4.36 for constant surface heat flux (Cengel 2003). For fully developed turbulent cooling flow in pipe, Nusselt number is given by Dittus and Boelter (1930) as;

$$Nu = 0.023Re^{0.8}Pr^{0.3} \quad (2)$$

Total heat transfer is given as

$$\dot{Q} = hA_s[T_{bm} - T_s] = \dot{m}C_p[T_{in} - T_{out}] \quad (3)$$

From equation (3), Constant surface temperature is given as

$$T_s = T_{bm} - \frac{\dot{m}C_p[T_{in} - T_{out}]}{hA_s} \quad (4$$

Where

T_{bm}, is the bulk mean fluid temperature of the fluid, and is given as: $\frac{1}{2}(T_{in} + T_{out})$ (5)

h, the internal convective heat transfer coefficient and is given as

$$\frac{k \times Nu}{D_i} \quad (6)$$

ṁ, the mass flow rate of the fluid given as

$$\pi \times R_e \times \mu \times \frac{D_i}{4} \quad (7)$$

A_s, the surface heat transfer area and is given as

$$\pi \times D_i \times L \quad (8)$$

R_e is the Reynolds number

P_r is the Prandtl number

C_p is the heat capacity

D_i is the internal diameter of the pipe

L is the length of the pipe

2.4 External Natural Convective Heat Transfer

A variety of theoretical expressions, graphical correlations and empirical equations have been developed to represent the coefficients for natural convection heat transfer from pipe. These studies were mostly focused on geometrical parameters of the pipe, such as diameter, orientation, length, as well as, directions. Some dimensionless numbers are important in natural convective heat transfer analysis for the determination of convective heat transfer coefficient. These dimensionless numbers are defied as follow;

The Nusselt number (Nu) is the ratio of convection heat transfer to the fluid conduction heat transfer under the same conditions.

$$Nu = \frac{Q_{convection}}{Q_{conduction}} = \frac{h}{k/D} = \frac{hD}{k} \quad (9)$$

Prandtl number (Pr) is the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity.

$$\mathrm{Pr} = \frac{\mu C_p}{k} = \frac{\rho v C_p}{k} = \frac{v}{k/\rho C_p} = \frac{v}{\alpha} \quad (10)$$

Grashof number (Gr) is the ratio of the buoyancy force to the viscous force acting on the fluid.

$$Gr = \frac{g\beta(T_s - T_\infty)D^3}{v^2} \quad (11)$$

Rayleigh number (Ra) is defined as the product of Grashof number, and the Prandtl number.

$$Ra = Gr \times \mathrm{Pr} \quad (12)$$

Where

g is the gravitational acceleration, m/s^2

ß is the coefficient of volume expansion, 1/K (ß = 1/T for ideal gases)

T_s is the temperature of the surface, °C

T_∞ is the temperature of the fluid sufficiently far from the surface, °C

L_c is the characteristic length of the geometry, m

v is the kinematic viscosity of the fluid, m^2/s

An extensive review on natural convective heat transfer from horizontal cylindrical pipe has been done by Boetcher (2014), who investigated the temperature boundary condition as well as heat flux boundary conditions by examining both early investigators and modern developments and found out that at small Rayleigh numbers, the heat transfer from a horizontal cylinder behaves like a line heat source. For larger Rayleigh numbers, i.e., $10^4 \leq \mathrm{Ra} \leq 10^8$, the flow forms a laminar boundary layer around the cylinder. At even higher Rayleigh numbers, it is expected that the flow becomes turbulent.

Faghri et al. (2010) stated that the boundary layer thickness around a horizontal cylinder when the wall temperature is higher than the ambient temperature, is a function of the angle θ (θ=0^o is at the bottom of the cylinder). The local Nusselt numbers along the surfaces of a cylinder obtained by Merk and Prins (1945) are shown in Fig. 2.1a.

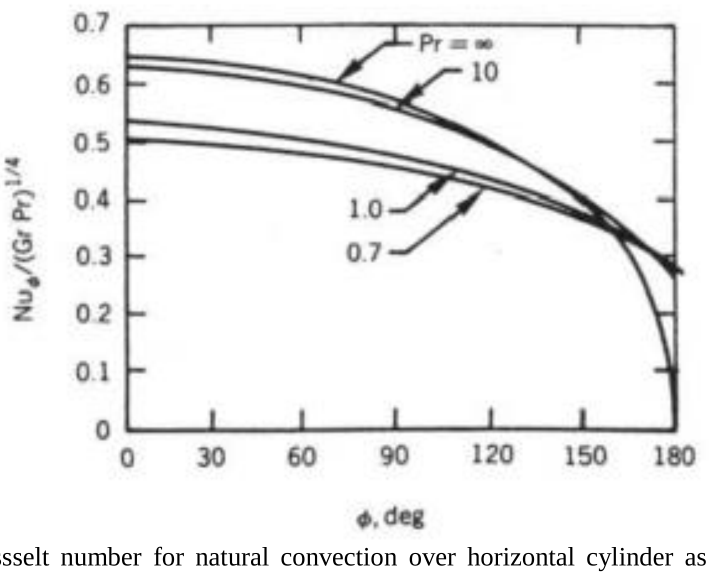


Fig. 2.1a: Local Nussselt number for natural convection over horizontal cylinder as giving by Merk and Prins, 1945) (Source: Faghri et al. 2010)

Merk and Prins (1945) suggested the empirical correlation for natural convection over a horizontal cylinder to be:

$$\overline{Nu} = C{Ra_D}^{1/4} \quad (13)$$

Where the characteristic length is the outside diameter of the cylinder and the constant C is given for Pr 0.71 as 0.436.

According to Faghri et al., the equation of Churchill and Chu (1975) covers all Prandtl number and Rayleigh number between 0.1 and 10^{12}

$$\overline{Nu} = \left\{0.6 + \frac{0.387{Ra_D}^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2 \quad (14)$$

Again, according to Cengel (2003), the rate of convection heat transfer ($\dot{q}_{conv}$) is discovered to be related to the temperature difference and is suitably expressed by Newton's law of cooling as;

$$\dot{q}_{conv} = h(T_s - T_\infty)(W/m^2) \tag{15}$$

or

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)(W) \tag{16}$$

Where
h is the convection heat transfer coefficient, W/m^2 °C
A_s is the heat transfer surface area, m^2
T_s is the temperature of the surface, °C
T_∞ is the temperature of the fluid sufficiently far from the surface, °C

Heat is being transferred from the surface of the pipe to air layer adjacent to the surface by pure conduction because of the no-slip and no-temperature-jump conditions. At this point conductive heat transfer is the same as convective heat transfer, and can be expressed as;

$$\dot{q}_{conv} = \dot{q}_{cond} = -k_{air}\frac{\partial T}{\partial r}|_{r=R_o}(W/m^2) \tag{17}$$

By equating the two expressions (15) and (17) for the heat flux to obtain the convective heat transfer coefficient which vary along the surface (or θ-) direction.

$$h = \frac{-k_{air}\frac{\partial T}{\partial r}|_{r=R_o}}{T_s - T_\infty} = \frac{\dot{q}_{cond}}{T_s - T_\infty}(W/m^2 \,.\,℃) \tag{17}$$

The average or mean convection heat transfer coefficient for the pipe surface is obtained by suitably averaging the local convection heat transfer coefficients over the entire pipe surface. McAdams (1954) gave a simplified equation that relates natural convection average heat transfer coefficient to ratio of temperature change and external diameter for horizontal pipe in air at atmospheric pressure for laminar flow (Ra $\leq 10^9$) conditions as;

$$\bar{h} = 1.32\left[\frac{\Delta T}{D}\right]^{0.25} \tag{18}$$

3.0 MATERIALS AND METHODOLOGY

3.1 COMSOL Multiphysics Software

The Multiphysics interface in the Software is used to compute physical fields like velocity, temperature, pressure or electromagnetic fields. The Conjugate Heat Transfer, Laminar Flow Multiphysics which combines the Heat Transfer in Solids and Laminar Flow interfaces, is used to simulate the coupling between heat transfer and fluid flow. The Non-Isothermal Flow Multiphysics coupling is automatically added. It couples the heat transfer and flow interfaces and provides options to include flow heating in the model. The fluid properties may depend on temperature. Models can also include heat transfer in solids as well as surface-to-surface radiation and radiation in participating media, with the Heat Transfer Module. The physics interface supports low Mach numbers (typically less than 0.3), as well as non-Newtonian fluids. Stationary modelling and time-domain modelling are supported in 2D, 2Daxi and 3D.

Hot water is allowed to flow through a metal pipe of length (L), internal and external diameters (D_i, D_o) of a known thickness (th) with a specified mass flow rate (ṁ). The pipe is placed in the air at a room temperature. Since the temperature of the outer fluid (air) is less than the inner fluid (hot water), heat is transferred by convection inside the pipe to the internal pipe surface then from the pipe internal surface to the exterior surface by conduction and from the pipe exterior surface to the air by natural convection. The schematic figure of the problem is shown in figure 3.1.

Hot Water flows through the inlet at T_{in} = 90°C and leaves at T_{out} = 80°C. The bulk mean temperature of the hot water as given in equation (5) is 85℃.

3.2 Problem Physics and Modelling

From the problem description in section 3.1, the heat transfer from the flowing fluid to internal surface of the pipe is modelled by forced convection, the heat transfer from internal surface of the pipe to external surface is modelled by conduction, while heat transfer from the external surface of the pipe to the ambient air is modelled by natural convection around horizontal pipe. The equations that govern the physics described are given in equations (19a) to (21d).Problem statement above was divided into four cases. For laminar internal flow Reynolds number was chosen to be 500, and 2×10^4 for turbulent internal flow. These four cases are summarized in table 3.1.

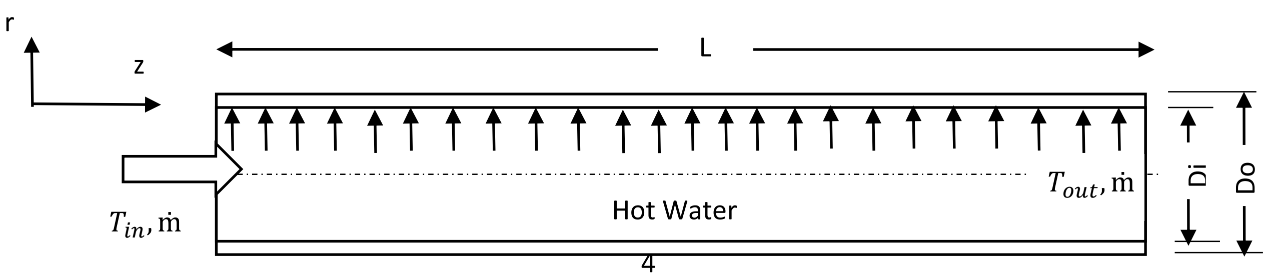


Fig. 3.1a: Problem description showing the internal convective flow

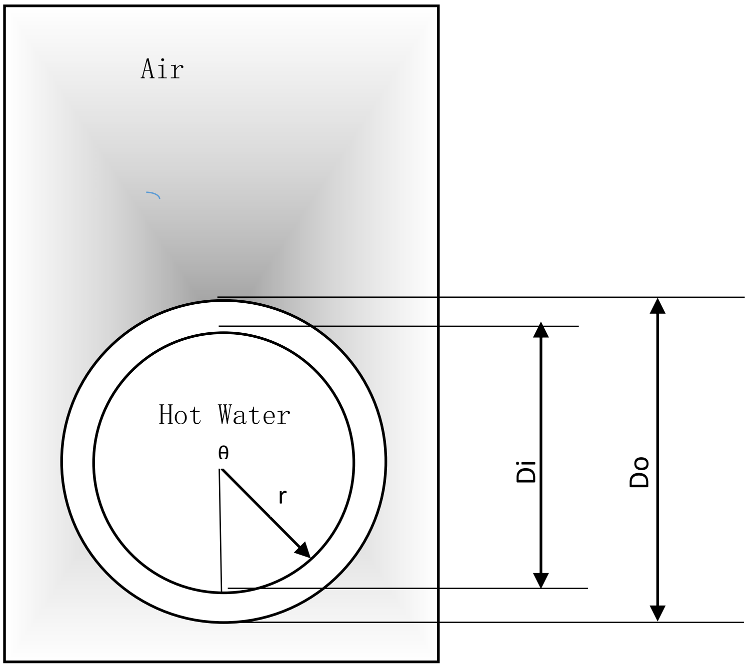


Fig. 3.1b: Problem description showing conduction in pipe and external convection

3.3 Internal Convection Governing Equations

$$Continuity: \quad \frac{\partial v_z}{\partial z} + \frac{1}{r}\frac{\partial (v_r)}{\partial r} = 0 \tag{19a}$$

$$r - momentum: \quad v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right) + \frac{\partial^2 v_r}{\partial z^2}\right] \tag{19b}$$

$$z - momentum: \quad v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{\partial^2 v_z}{\partial z^2}\right] \tag{19c}$$

$$Energyequation: \quad v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial x^2}\right] \tag{19d}$$

$$Conductionthroughthepipematerial: \quad \frac{1}{r}\frac{\partial}{\partial r}\left[kr\frac{\partial T}{\partial r}\right] = 0 \tag{20}$$

3.4 External Natural Convection in the Air Surrounding the Pipe:

$$Continuity: \quad \frac{1}{r}\frac{\partial (v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = 0 \tag{21a}$$

$$r - momentum: \quad v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{{v_\theta}^2}{r} = \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + g\beta(T - T_\infty) \tag{21b}$$

$$\theta - momentum: \quad v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{v_r v_\theta}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + \frac{\mu}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} - \frac{v_\theta}{r^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] \tag{21c}$$

$$Energyequation: \quad v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} = \frac{k}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right] \tag{21d}$$

Table 3.1: Summarized problem approaches

Case	Physics	Reynolds number (Re)	Internal flow situation	Boundary condition
1	External free convection only	500	Fully developed laminar flow	Constant wall temperature
2	External free convection only	2×10^4	Fully developed turbulent flow	Constant wall temperature
3	External free convection and conduction in the pipe material	500	Fully developed laminar flow	Constant wall temperature
4	External free convection and conduction in the pipe material	2×10^4	Fully developed turbulent flow	Constant wall Temperature

3.5 Geometrical Considerations of Pipe and its Properties

3.5.1 Entry Length

Entry length is the region from the pipe inlet to the point at which the thermal or velocity boundary layer merges at the centre of the pipe to become fully developed. It is referred to as thermal entry length when the thermal boundary layer develops and reaches the centre of the pipe, while it is hydrodynamic entry length when the velocity boundary layer develops and reaches the centre of the pipe.

The hydrodynamic entry length according to Cengel (2003), is usually taken to be the distance from the tube entrance where the friction coefficient reaches within about two percent of the fully developed value. They further cited Kays and Crawford (1993) and Shah and Bhatti (1987) that in laminar flow, the hydrodynamic and thermal entry lengths are given approximately as:

$$L_{hlaminar} \approx 0.05 R_e D \tag{22}$$

$$L_{tlaminar} \approx 0.05\, R_e P_r D \approx P_r L_{hlaminar} \tag{23}$$

While in turbulent flow, both hydrodynamic and thermal entry lengths are given as:

$$L_{h,turb} = L_{t,turb} \geq 10D \tag{24}$$

Similarly, Faghri et al. (2010), gives the hydrodynamic entry length for turbulent flow to be

$$\frac{L_{hturb}}{D} \geq 0.625\, {R_e}^{0.25} \tag{25}$$

Where

L_h is the hydrodynamic entry lengths

L_t is the thermal entry lengths

turb = turbulent flow

laminar = Laminar flow

3.5.2 Length of the Pipe

Length is selected based on the entrance (hydrodynamic and thermal) lengths of the flowing hot water from inlet. The entrance length in terms of the internal diameter of the pipe is given in equations (22) and (23) for laminar flow (Re < 2300) and (24) and (25) for turbulent flow (Re > 10000). The length is selected to be greater than both the thermal and hydrodynamic length in order for the fully developed assumption to be satisfied. The entrance lengths are calculated from equations (22) to (25) and the calculated values with Prandtl number at 85ºC are shown in table 3.2. From the table, the length of pipe that can satisfy the fully developed assumption must be greater than $52D_i$. Therefore, the length of the pipe is selected to be $70D_i$ (i.e. 70 times the internal diameter of the pipe). For a standard carbon-steel pipe of schedule 120, one-inch nominal pipe size (NPS); Outer diameter, D_o = 1.315 in (33.40 mm); Schedule 120 thickness, th = 0.2 in (5.08 mm); Internal diameter, $D_i = D_o - 2th$ = 0.915 in (23.24 mm); Therefore, length of the pipe = $70D_i$ = 64.05 in (1627 mm or 1.627 m).

3.5.3 Thermal Properties of the Pipe

The carbon-steel pipe material thermal properties are evaluated at 300K. The properties are summarized in table 3.2.

3.6 Thermal Properties of Water

The thermal properties of the flowing water are evaluated at bulk mean fluid temperature of 85℃. The properties are listed in table 3.3.

Table 3.2: Entrance lengths calculated values

Pr(85℃) = 2.08	Hydrodynamic	Thermal
Re	$[L_H]_{Cengel}$ $[L_H]_{Faghri}$	$[L_T]_{Cengel}$ $[L_T]_{Faghri}$
500	$25D_i$ $25D_i$	$52D_i$ $52D_i$
2×10^4	$10D_i$ $7.4D_i$	$10D_i$ $10D_i$

Table 3.3: Pipe Material Properties

Parameters	Definitions	Values	Units
Geometry:			
D_i	Internal Diameter	23.24	mm
D_o	Outer Diameter	33.40	mm
L	Length	1.627	m
Thermal:			
K	Thermal Conductivity	60.5	W/m-K
C_p	Specific Heat Capacity	434	J/kg-K
P	Mass Density	7854	kg/m^3
α	Thermal Diffusivity	17.7×10^6	m^2/s

Source: Incropera, Frank P. and DeWitt, David P. (1996)

3.7 Grashof and Rayleigh Numbers

The Grashof numbers of the air flow in the first four cases are calculated using equation (10) and the Rayleigh number by equation (11). The calculated value for the Grashof numbers are 1.75×10^5, 1.54×10^5, 1.74×10^5 and 1.45×10^5 for case 1, 2, 3 and 4 respectively; while Rayleigh numbers are 1.26×10^5, 1.12×10^5, 1.26×10^5 and 1.05×10^5 for case 1, 2, 3 and 4 respectively. Since the value of Rayleigh numbers in all the four cases is less than the critical values (i.e. $10^4 < Ra < 10^9$), the flow regime is computed using laminar fluid flow interfaces for the four cases.

3.8 Computational Details

A finite element simulation was implemented using COMSOL Multiphysics for the flow of the two fluids and both convective and conductive heat transfer. Two different 2D models were used to depict the external and internal pipe flows. The type of physics applied was conjugate heat transfer that involves fluid flow and heat transfer. This allowed for definition of not only the fluid flow parameters but also the heat transfer parameters in fluid. The first step solved for natural heat convection in the external air only for cases one and two. The second step solved for heat conduction through the pipe material and a convective heat transfer in the external air for cases three and four. The last step combined all the results in case one to four to solve for the forced internal convection of hot water inside the pipe.

3.8.1 Geometry Description

The air domain is considered rectangular having a width of $3D_o$ and a height of $10D_o$. The horizontal pipe is represented as a circular cut in the centre of the rectangle as shown in figure 3.2a for case 1 and 2, and figure 3.3a for cases 3 and 4. In cases 5 and 6, the pipe is represented by a rectangle having D_i as the width and L as the length as shown in figure 3.4a.

3.8.2 Initial and Boundary Conditions

The boundary condition that was used to simulate the cases is the constant surface temperature and velocity of the air. The ambient temperature of the air is at 25ºC, while the constant temperature along the pipe surface was calculated for the four cases from equations (2) to (8) and is given in table 3.5. The velocity of the air around the pipe was assumed to be zero, i.e. at rest and no slip boundary conditions are imposed on the surface of the pipe.

Table 3.4: The Properties of Hot Water at 85ºC

Parameters (85 ºC)	Definitions	Values	Units
ρ	Mass Density	968.1	kg/m^3
C_p	Specific Heat Capacity	4201	J/kg-K

K	Thermal Conductivity	0.673	W/m-K
M	Dynamic Viscosity	0.333×10^{-3}	kg/m.s
P_r	Prandtl Number	2.08	---

Source:Sengers, J. V. and Watson, J. T. R. (1986)

Table 3.5: Boundary Condition Constant Surface Temperature

Case	Nu	H ($W/m^2 K$)	ṁ (kg/s)	A_s (m^2)	T_s (°C)
1 & 3	3.66	106	3.04×10^{-3}	0.1188	74.86
2&4	79.06	2289.47	0.12156	0.1188	66.22

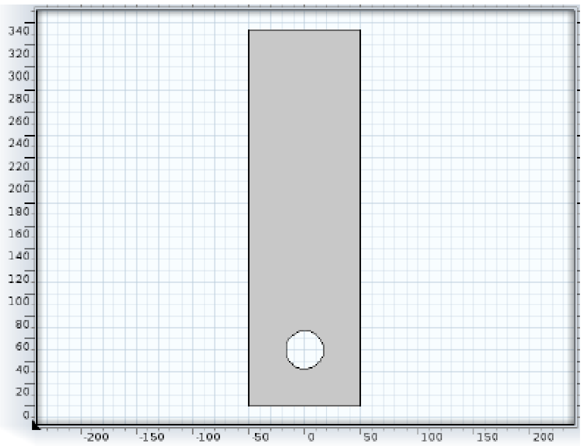


Figure 3.2a: Geometry of the model domain for case 1 and 2

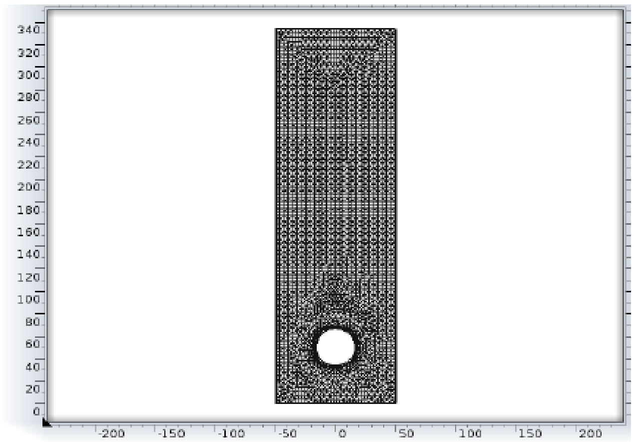


Figure 3.2b: Mesh distribution of the model domain for case 1 and 2

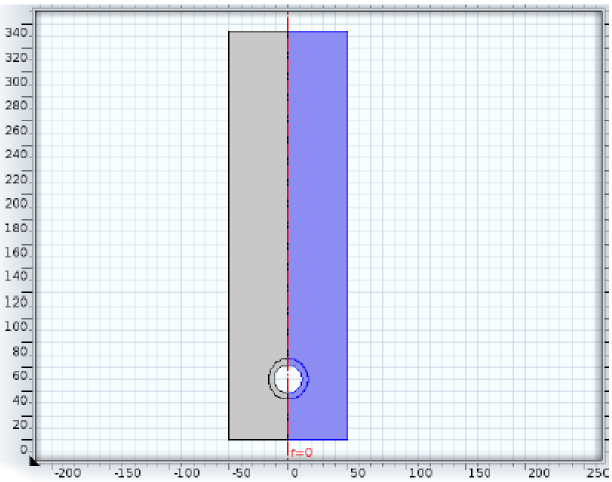


Figure 3.3a: Geometry of the model domain

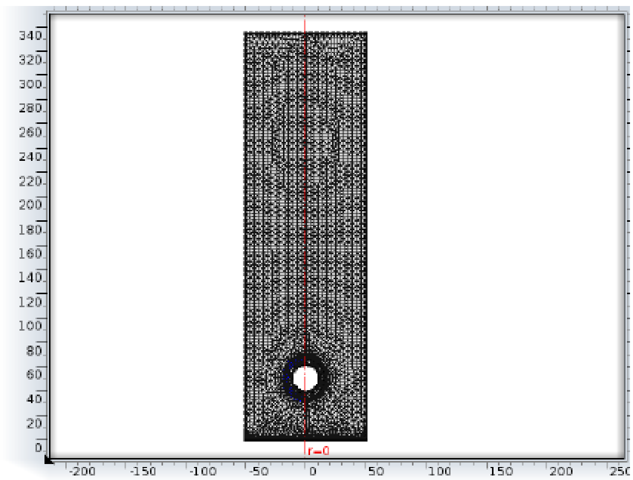


Figure 3.3b: Mesh distribution of the model for for case 3 and 4 domain for case 3 and 4

3.8.3 Properties of External Air

The properties of air were defined in COMSOL using its material browser. The basic properties used were dynamic viscosity, specific heat capacity at constant pressure, density and thermal conductivity. These properties were computed by the software at different iterations before the convergence.

3.8.4 Mesh Generation

The computational domain was discretized into finite triangular elements with sequence type of physics control mesh and element size of extra fine. The complete mesh consists of 9716 elements for case 1 and 2 (fig. 3.2b), 9824 elements for case 3 and 4 (fig. 3.3b)

4.0 RESULTS AND DISCUSSION

All the cases were solved using finite element Lagrange formulation nonlinear fully coupled with maximum number of 50 iterations. It acquired 4, 6, 16 and 19 minutes for case 1, 2, 3 and 4 respectively on a CPU of speed 2.20GHz with 4GB ram. The solutions were obtained in terms of temperature, velocity and density distribution in the air and temperature and velocity distribution inside the pipe. The local convective heat transfer coefficient and local Nusselt number distribution at the surface of the pipe were also obtained. A simplified relation for average heat transfer coefficient and average Nusselt number for horizontal pipe in air for laminar condition were obtained and found a good agreement with literature.

4.1 Temperature Distribution in the Air

Figure 4.1 a – d show the temperature distribution in the air for the first four cases. It can be seen from the plots that the thermal boundary layer starts to develop from the bottom of the pipe, increasing in thickness along the circumference and forming a rising plume at the top. As we move up from the top of the pipe, the temperature of the air drops back to the ambient temperature at 25ºC. It can be seen that temperature in case 1 and 2drop more rapidly than that of case 3 and 4 due to high conductive heat flux from the pipe material used in case 3 and 4.

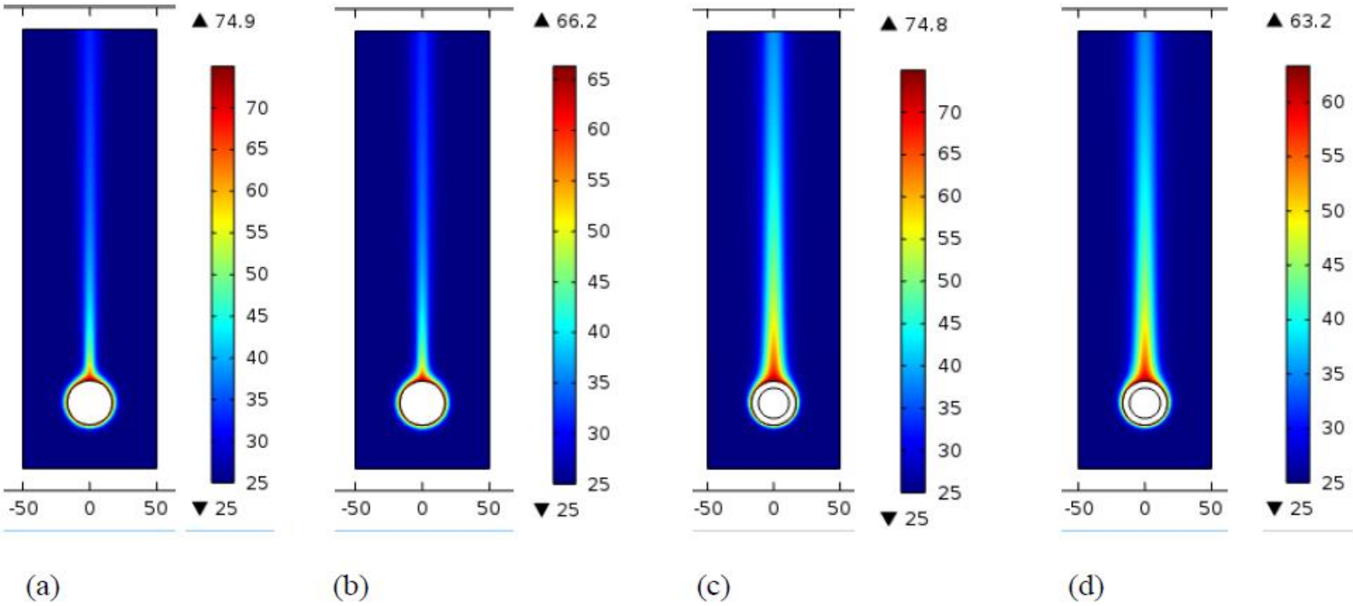


Fig. 4.1: Temperature distribution in the air around the pipe; (a) case 1 (b) case 2 (c) case 3 and (d) case 4.

4.2 Temperature Distribution in the Pipe Material

Figure 4.2 a show the temperature distribution in the pipe material due to conduction in case 3 and 4. It can be inferred from the plots that the temperature drop in case 3 is about 0.1% and 3.1% in case 4 due to high heat transfer rate in case 4.

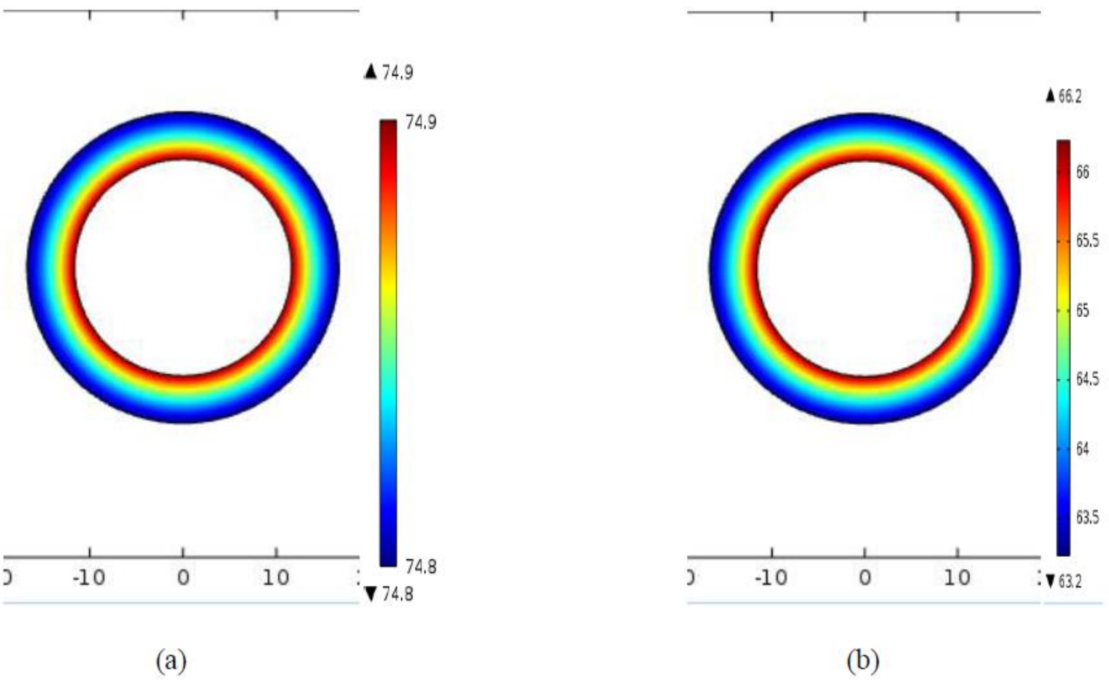


Fig. 4.2 a: Temperature distribution in the pipe material due to conduction (ºC) (a) case 3, (b) case 4

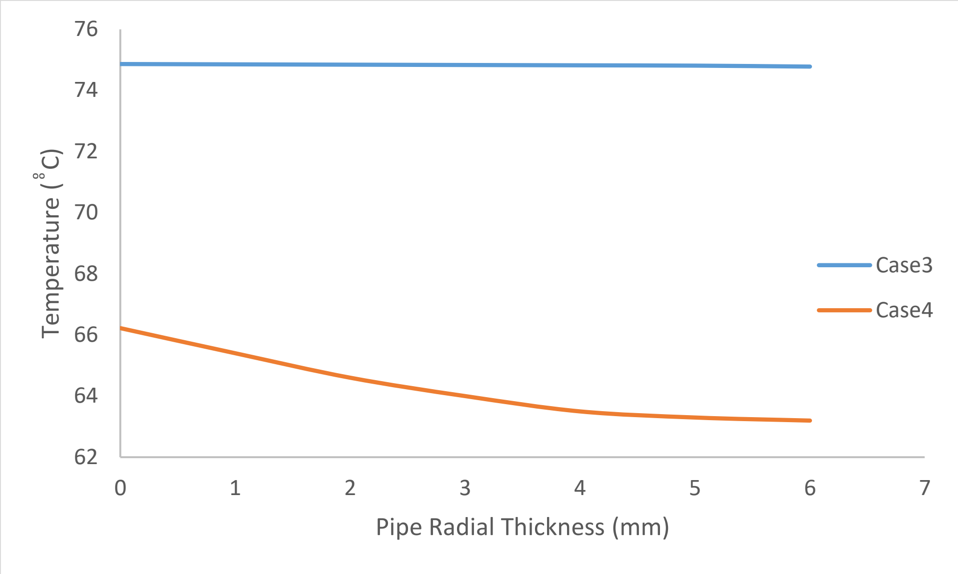


Fig. 4.2b: Temperature graph at different points along the radius of the pipe material

4.3 Velocity Distribution

Figure 4.3a – d show the velocity distribution in the air in the first four cases. The velocity of the air is increasing due to the convection currents which are formed due to decreasing in air density near the pipe. It is interesting to note the stagnation point at the top of the pipe that is formed due to the mixing air from both sides of the pipe forming thermal insulation above the pipe. The velocity of the air around the pipe surface is very low due to no-slip condition where hydrodynamic boundary layer starts to grow. It can be inferred from the figures that velocity of air in cases 3 and 4 is higher than that of case 1 and 2 due to highly conductive layer of pipe from.

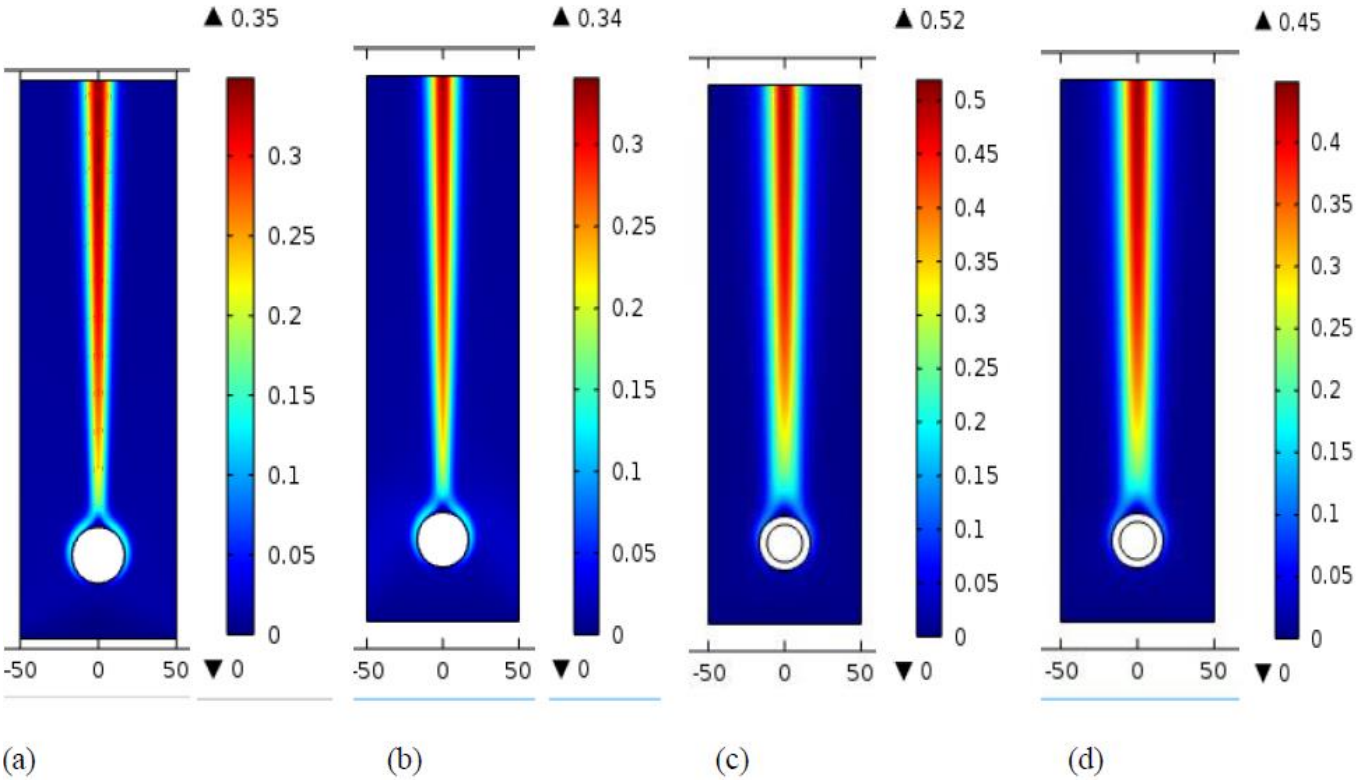
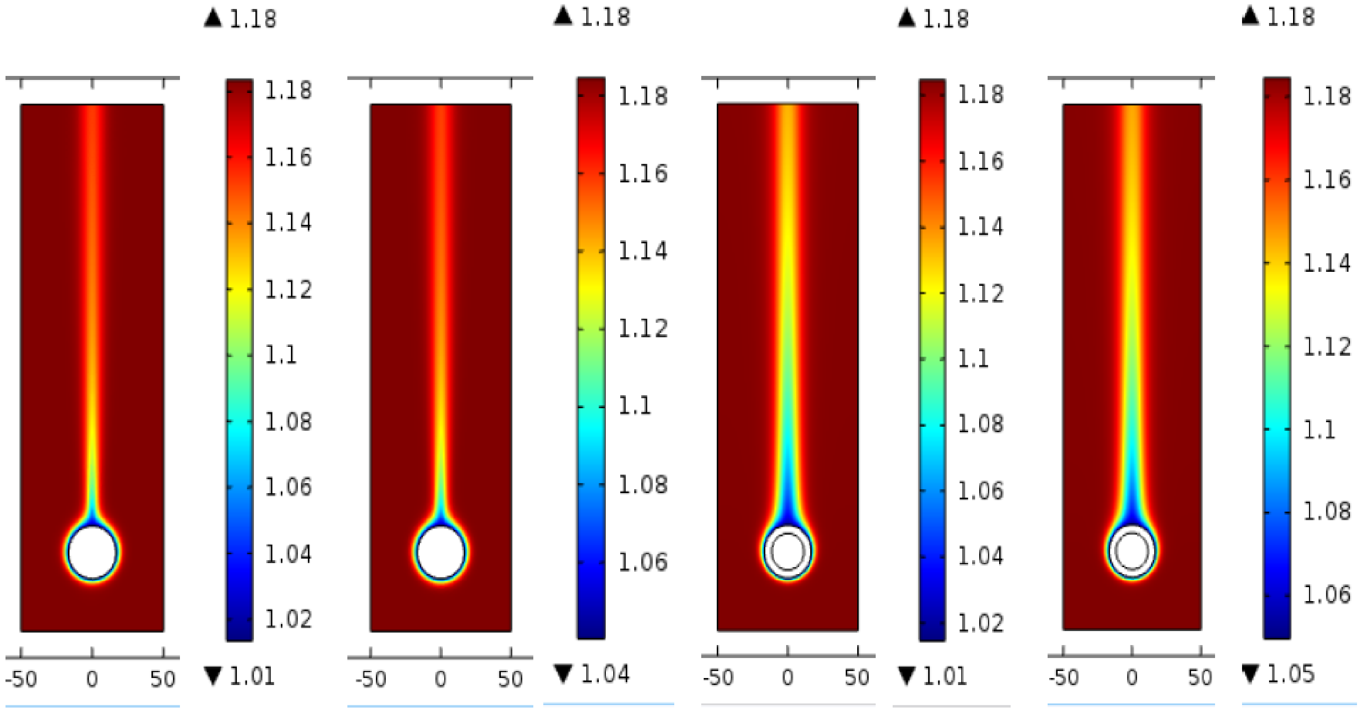


Fig. 4.3: Velocity distribution in the air around the pipe (in m/s) (a) case 1 (b) case 2 (c) case 3 and (d) case 4

4.4 Density Distribution

Figure 4.4 a – d show the density distribution in the air in all the four cases. Due to temperature difference between air and the pipe, the air density near the pipe surface becomes less dense and upsurges. The surrounding cooler air then moves to replace the less dense air. This cooler air is then heated and the process continues, forming a convection current. Also the presence of gravity makes the less dense air to rise, due to buoyancy effect, a result of differences in air density obviously observed in the plots. The density of air around the surface of the pipe in case 1 and 3 drops by 14.4% as compare to that of case 2 and 4 which drops by 11%. This is due to the fact that the surface temperature of the pipe in case 1 and 3 is more than that of case 2 and 4

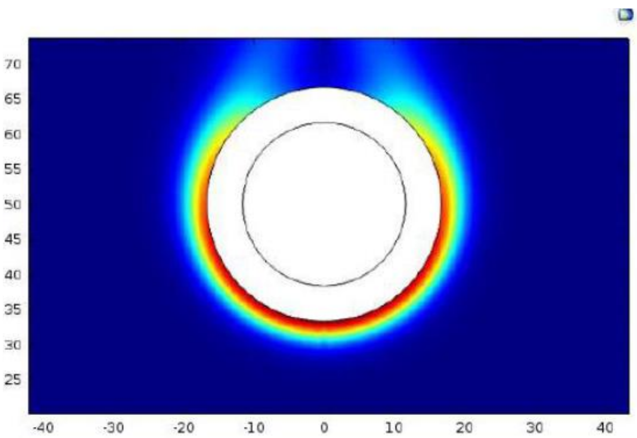


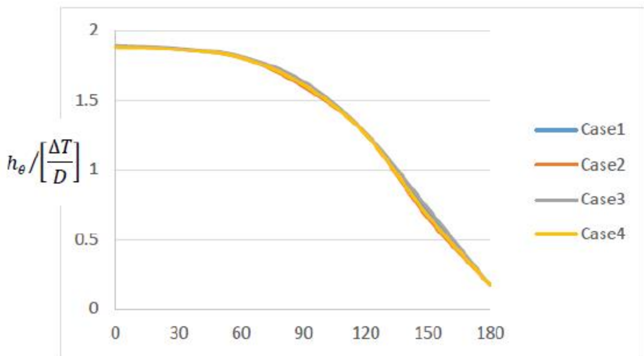
(a) (b) (c) (d)

Fig. 4.4: Density distribution in the air around the pipe (in kg/m^3); (a) case 1 (b) case 2 (c) case 3 and (d) case 4

4.5 Heat Transfer Coefficient

The immediate air that is in contact with the pipe surface have the same temperature as the pipe surface due to no-temperature-jump condition. Since the air layer adjacent to the pipe is not moving (no-slip) and has the same temperature as that of the surface (no-temperature jump), the heat transfer from the pipe surface to the adjacent air is purely by conduction. Therefore the local heat transfer coefficient is acquired around the pipe surface as shown in figure 4.5a. It can be seen from the graph that in all the four cases, the local heat transfer coefficient declines along the pipe from the bottom (θ=0°) of the pipe to the top (θ=180°) due to decrease in local heat flux along the trend. The normalized local heat transfer coefficient for the first four cases were shown in figure 4.5b. A simplified equation that relates natural convection average heat transfer coefficient to ratio of temperature change and external diameter for horizontal pipe in air at atmospheric pressure for laminar flow (Ra = 10^5) conditions was obtained to be $\bar{h} = C\left[\frac{T_s - T_\infty}{D}\right]^n$ where constant C and n were found using Microsoft excel to be 1.314 and 0.25 respectively which establish a good agreement with the work of McAdams (1954), equation (18).





(θ=0° to 180°)

Fig. 4.5a: Local heat transfer coefficient around the pipe surface (W/m^{2}K)

Fig.4.5b: Normalized local heat transfer coefficient around the pipe surface

4.6 Nusselt Number

As expected, by moving from the bottom (0°) of the pipe to the top (180°), the local Nusselt number shows a decreasing trend as shown in figure 4.6a due to decrease in local heat transfer coefficient. The Nusselt number was normalized by fourth-root of Rayleigh number as shown in figure 4.6b. A simplified equation that relates natural convection average Nusselt number to Rayleigh number for horizontal pipe in air at atmospheric pressure for laminar flow (Ra = 10^5) conditions was obtained to be $\overline{Nu} = CRa^n$ where constant C and n were found to be 0.44 and 0.25 which found a good agreement with the work of Merk and Prins (1945), equation (13).

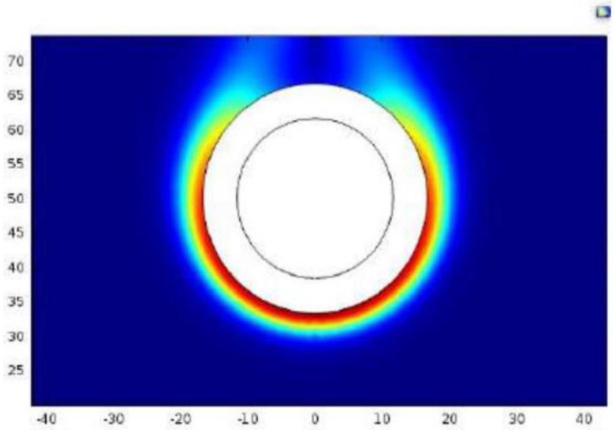


Fig. 4.6a: Local Nusselt number around surfaces

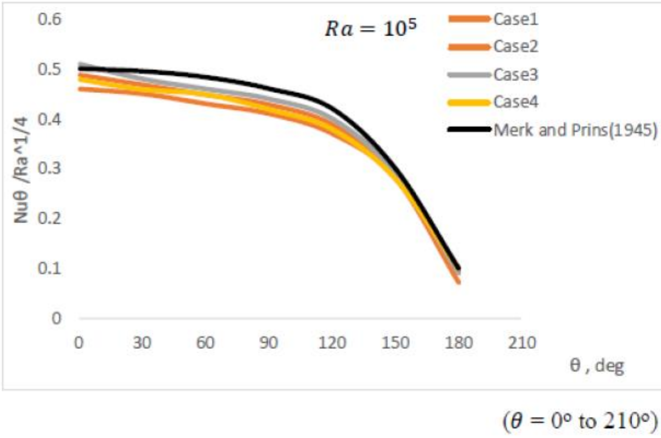


Fig. 4.6b: Normalized Nusselt number the pipe the pipe surface

4.7 Internal Forced Convection inside the Pipe

Figure 4.7 and 4.8 show the velocity and temperature distribution inside the pipe for case 5 and 6. As can be seen in figure 4.7a the hydrodynamic entrance length is about $25D_i$ while in 4.7b is about $7D_i$ because the hydrodynamic length in l ℃ flow is more than that of turbulent flow in pipe which confirms what Cengel (2003) and Faghri et. al. (2010) reported. The velocity in figure 4.7a is fully developed at about the one-third of the length of the pipe while is about one-tenth of the length in figure 4.7b. The average velocity of case 5 (fig. 4.7a) is about 0.01m/s and 0.34m/s.

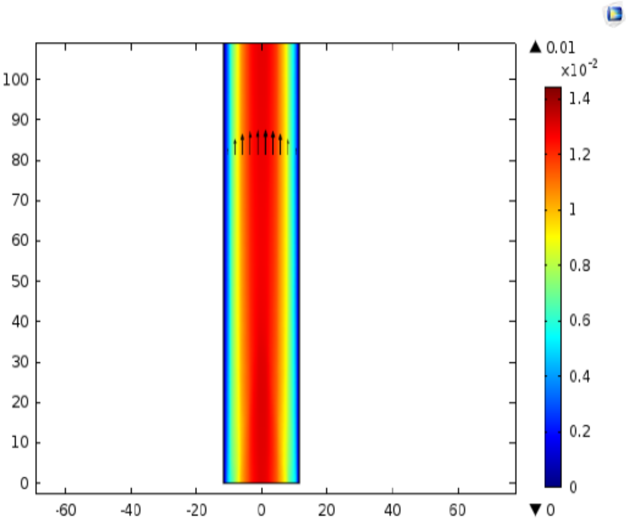


Fig. 4.7a: Laminar Velocity Profile with Length (Case 3)

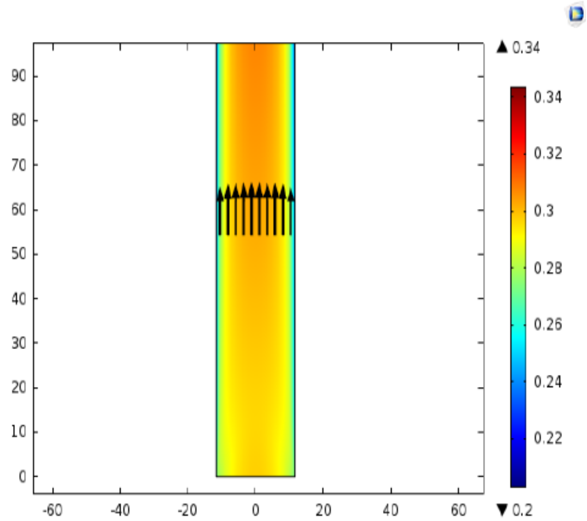


Fig. 4.7b: Turbulent Velocity Profile Hydrodynamic Entry with Hydrodynamic Entry Length (Case 4)

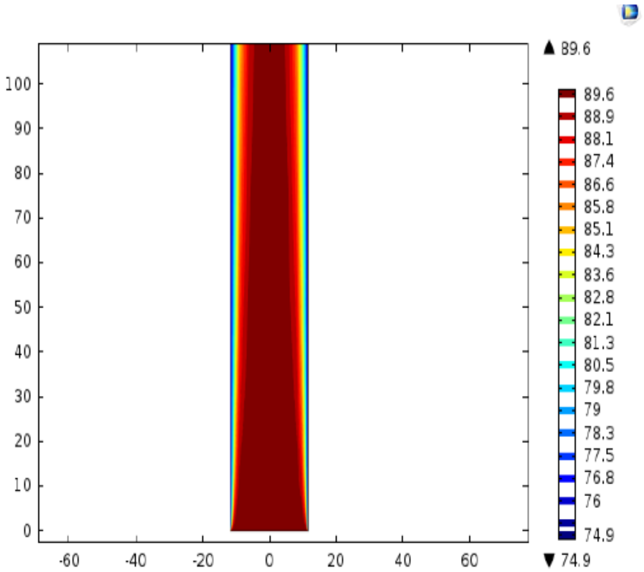


Fig. 4.8a: Laminar Temperature Profile with Thermal Entry Length (Case 3)

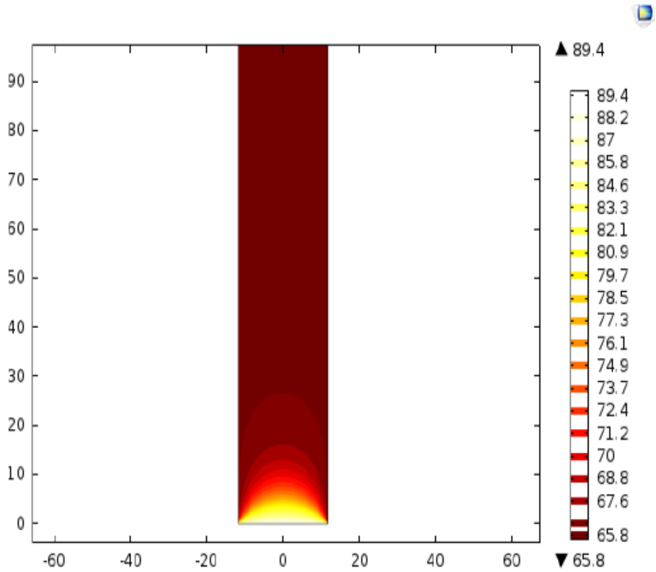


Fig. 4.8b: Turbulent Temperature Profile with Thermal Entry Length (Case 4)

From figure 4.8a, the thermal entrance length is about $50D_i$ while in 4.8b is about $10D_i$ due to the same reason stated above.

5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In this study, steady state natural convection in the air around the pipe and steady laminar and turbulent forced convection of hot water flowing inside the pipe using COMSOL Multiphysics as the design modeler, have been investigated for both convective heat transfer and temperature distribution The results were presented in terms of temperature, velocity and density distributions in the air around the pipe and temperature and velocity profile in the hot water flowing inside the pipe. In cases 1 to 4, the density drop in air is due to the temperature difference in the air and by the presence of gravitation, the lighter air upswings while the denser air drops. The velocity of the air is increasing due to the convection currents caused by buoyancy effect due to temperature difference. The local heat transfer coefficient was also presented for the circumference distance from the bottom to the top of the pipe. The values of the local heat transfer coefficients were normalized by ratio of the temperature difference and the external diameter of the pipe. Hydrodynamic and thermal entrance effect on the hot water flowing inside the pipe were also computed and established what is in the literature.

5.2 Recommendations

This work has been limited to studying the steady laminar natural convection taking place in the air surrounding a horizontal and straight pipe through which single-phase hot water is flowing using COMSOL Multiphysics software. This work may be extended to the following studies:

i. Transient natural convection phenomena in the air may be invoked through temporal temperature or flow rate variations of the internal pipe flow,
ii. Steady turbulent natural convection in the air surrounding the pipe may be simulated with two-phase flow of hotter condensing steam inside it,
iii. Steady turbulent natural convection in the air surrounding the pipe may be simulated with single-phase flow of hotter oil inside it, and
iv. Steady natural convection in the air surrounding an inclined or curved pipe may be simulated with single-phase flow of hot water inside it.

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